PROCEEDINGS OF THE MATHEMATICS IN INDUSTRY STUDY GROUP

2020

Mathematics in Industry Study Group South Africa
MISGSA 2020

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The seventeenth Mathematics in Industry Study Group (MISG) in South Africa was held at the University of Zululand, Empangeni Campus, KwaZulu Natal, from Monday 13 January to Friday 17 January 2020.

The total number of registered participants at the MISG was sixty-six. There were twenty-five Academic Staff, thirty-six Graduate Students and five Industry Representatives. The invited guests were:

Neville Fowkes	University of Western Australia, Australia
Graeme Hocking	Murdoch University, Western Australia, Australia
Tim Myers	Centre de Recerca Matematica, Barcelona, Spain
Denis Ndanguza	University of Rwanda, Rwanda
Stephen Wilson	University of Strahclyde

Two PhD students visited from overseas

Joseph Daniel Field	University of Oxford
Oliver Whitehead	University of Oxford

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Sciences
Council for Scientific and Industrial Research (CSIR)
Durban University of Technology
Nelson Mandela University
North-West University
Rhodes University
University of Cape Town
University of Johannesburg
University of KwaZulu-Natal
University of Pretoria
University of South Africa (UNISA)
University of Stellenbosch
University of Venda

University of the Western Cape
University of the Witwatersrand
University of Zululand

The MISG was officially opened on Monday morning by Professor Kunene Dean of Science, University of Zululand.

The MISG followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2019. Six problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by an academic moderator and one or more student moderators. The role of the academic moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results which were reported. The MISG was officially closed at lunch time on Friday by Dr. Matadi Head, Department of Mathematical Sciences, University of Zululand.

The MISG was preceded by a Graduate Modelling Camp from Wednesday 8 January to Saturday 11 January 2020. The objective of the Graduate Modelling Camp is to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working collaboratively in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. Six problems were presented to the graduate students. The problems and the presenters were:

Detecting imbedded thin strips	Neville Fowkes University of Western Australia
Designing an online shop	Jeff Sanders African Institute for Mathematical Sciences and University of Stellenbosch
Phenology model for the jacaranda blossoms	Jennifer Fitchett University of the Witwatersrand
Green roofs for managing stormwater runoff	Anne Fitchett and Gideon Fareo University of the Witwatersrand
mathematical modelling of tornadoes	Thama Duba University of the Witwatersrand
Thermal plumes in Lake Kivu	David Mason University of the Witwatersrand

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

The sponsors of the Graduate Workshop and the MISG were:

- Hermann Ohlthaver Trust
- African Institute for Mathematical Sciences
- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences
- School of Computer Science and Applied Mathematics, University of the Witwatersrand
- University of Zululand

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

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PROBLEM STATEMENTS

Problem 1. Models for honeybee arrival and blossom phenology

Industry: Climate change

Industry Representative: Jennifer Fitchett, School of Geography, Archaeology and Environmental Studies, University of the Witwatersrand

Problem statement

Phenology refers to the timing of annually recurrent biological events and their biotic and abiotic drivers. These phonological events are triggered by the change in seasons, as temperatures change and rainfall begins or terminates. Under climate change, the timing of these events is shifting. In many instances, events such as blossoming that marked the beginning of spring are now occurring in late winter. Each species, however, responds to a discrete selection of abiotic forces. For one species, the trigger may be the occurrence of daytime temperatures warmer than 20 degrees Celsius; for another it may be the timing of first rainfall. This results in a progressive mismatch between pollinators and predators and prey, as one species will arrive/hatch/emerge from hibernation long before its food supply is available. In this problem we are considering the plight of honeybees under climate change, as a result of the progressive advance of blossoming in South Africa. Models for honeybee arrival and blossom phenology will need to be developed and compared to calculate the threat of mismatch.

Problem 2. Modelling the turbulent flow in Lake Kivu

Industry: Energy

Industry Representative: Denis Ndanguza, University of Rwanda, Rwanda

Problem statement:

Turbulence is an irregular motion characterized by chaotic changes in pressure and flow velocity and which in general makes its appearance in fluids, gaseous or liquid. Generally, this is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates so that statistically distinct average values can be discerned. It is in contrast to a laminar flow, which occurs when a fluid flows in parallel layers, with no disruption between layers. Turbulence is encountered in most flows in nature and industrial applications. Natural turbulent flows can be found in oceans, rivers, lakes and in the atmosphere, whereas industrial turbulent flows can be found in heat exchangers, chemical reaction, etc. Turbulence arises due to instability occurring at high Reynolds numbers. Turbulence modelling is essential in environmental flows, which comprise flows in rivers, estuaries, coastal seas and lakes. Previous researchers have shown that the Reynolds number in Lake Kivu is high and this is a sign of turbulence existence. Down to a certain depth, turbulence is caused by waves and currents generated by winds and eddies due to surface cooling. In Lake Kivu this mechanism happens between 60 and 70 m of the lake.

Modelling turbulence in Lake Kivu is therefore of essential importance to the simulation of flow, the temperature (turbulent movement can spread the temperature) and biological activity in the lake. Sometimes, the wide range of scales and apparently random nature of turbulent eddies make turbulence difficult to model and a wide range of turbulence modelling approaches can be developed. Based on this motivation the issue addressed here is to apply any technique in fluid dynamics to model the turbulence movement in Lake Kivu which is complex in terms of stratification and stability.

Problem 3. Juice holdup detection in a sugar cane diffuser

Industry: Sugar cane processing

Industry Representative: Richard Loubser, Sugar Milling Research Institute, University of KwaZulu Natal, Durban

Problem Statement

The process for extraction of sugar cane in most South African mills involves a counter current washing process in a diffuser.

The best results are achieved with maximum wetting or juice holdup in the cane bed. The diffuser has sight glasses along the length of the diffuser for the operators to judge how much juice is held up in the bed. The operators then make adjustments to diffuser settings to change juice circulation patterns to compensate for changes in juice quantity that they see in the sight glass.

The sight glass is in a recess so there is a gap between the fiber of the bed and the glass surface. What the operator sees is a level of juice between the bed and glass, If this is a true level in the bed it implies that the juice backs up from the diffuser screen to a height in the bed. Since juice flows down from the top of the bed and the diffuser screen at the lower boundary is perforated, the presence of a distinct liquid level is disputed. The level in the sight glass is often observed to drain away suddenly and then reestablish itself a short time later. The liquid level has not been observed in laboratory experiments using a cylindrical column or a rectangular tank with a static cane bed. This suggests that the level observed in full scale could be a boundary effect at the wall of the full-scale diffuser.

Since work is being done, using image analysis, to measure the position of the sight glass liquid level, it is important to know what this level represents. A model which relates the actual hold-up of juice in the bed, possibly including aspects such as the cane permeability, its variation, diffuser bed height, juice feed to the top of the bed and juice distribution history, is needed to assess the relationship between juice hold-up or cane bed saturation, and the juice level observed in the sight glass. This will help in the interpretation of the measured level in terms of the degree of holdup in the adjacent cane bed.

Problem 4. Image analysis of sugar cane preparation

Industry: Sugar cane processing

Industry Representative: Richard Loubser, Sugar Milling Research Institute, University of KwaZulu Natal, Durban

Problem Statement:

The process for extraction of sugar cane in most South African mills involves a counter current washing process in a diffuser.

The first step is to break the cane stalk into fine pieces to expose the sugar containing juice for extraction. This preparation of the cane is done using a hammer mill shredder. After preparation, the shredded cane is fed to a diffuser where it is extracted using a counter-current washing process. Best results are achieved with maximum contact between the percolating juice and cane. Too much juice, however, leads to flooding and uncontrolled mixing of the juice with an associated loss in extraction.

Percolation performance of the shredded cane in the diffuser depends on the degree of preparation. If the cane is underprepared, it is difficult to wash the sugar from the cane. Over-prepared cane will form a more densely packed bed in the diffuser with low permeability. The permeability influences juice flow patterns in the diffuser and low permeability can cause flooding of the diffuser.

The degree of preparation depends on the variety and growing conditions of the cane. The shredder is set to accommodate the average cane that is prepared. This leads to variation in the degree of preparation and hence the permeability of the cane bed.

Currently cane preparation is measured using an off-line process. It would be useful if the cane preparation could be measured in real time allowing continuous adjustments to be made to the shredder clearances to compensate for variability in the cane.

A good front-end engineer would look at the prepared cane and be able to judge the degree of preparation. Can machine vision be used to achieve the same result?

Data was collected during an experiment at the beginning of 2019. Samples of cane with differing levels of brown leaf were prepared and then tested to give parameters which were designed to express percolation and extraction characteristics of the cane. The cane samples were also photographed.

Four cane varieties were used. Subsamples of each were subjected to the following treatments:

- Leaf was burned off the cane stalk
- Leaf was stripped by hand
- Medium amount of leaf was left on cane
- High amount of leaf was left on cane

Each treatment was analysed in triplicate as follows:

- Percolation rate: A given mass of cane was loaded into a column and percolation rate (hence permeability) was measured
- Density: A given mass of cane was loaded into a tube and a fixed force applied to the cane with a plunger. After a fixed time, the volume was determined giving the density of the cane. (Density is strongly correlated to flow rate.)
- Displacement rate index: Cane was washed and the time constant for the mass transfer determined. Conductivity was used as an indicator of the mass transfer.
- Photograph: Three different photographs were taken of each batch of cane under similar lighting conditions.

The number of samples was therefore:

$$(4 \text{ varieties}) \times (4 \text{ treatment}) \times (3 \text{ replicates}) = 48 \ (48 \text{ photographs})$$

This is a preliminary study to determine if there is sufficient information in the photographs to determine whether it is feasible to use machine vision to extract the degree of preparation of the cane. If this shows a reasonable chance of success, techniques of taking photographs on a conveyor can be developed and more data collected for training a system.

Problem 5. Green roofs to mitigate the urban heat island

Presenter: Professor Anne Fitchett, Assistant Dean, Faculty of Engineering and the Built Environment, University of the Witwatersrand and Gideon Fareo, School of Computer Science and Applied Mathematics, University of the Witwatersrand

Problem statement

The Urban Heat Island is a phenomenon where the temperature of a region of a city (generally the inner city) is higher than the surroundings. This is caused by the prevalence of dense and dark coloured materials. The dense materials, such as concrete, absorb solar radiation in the daytime and store it until the evening, when the heat is released into the atmosphere by convection. Very dark material, such as asphalt, absorbs much more heat than lighter colours that reflect most of the solar radiation back into the atmosphere. The combination of these two types of materials cause the area of an inner city to be several degrees warmer than the surrounding areas, causing an up-swell of air that induces thunderstorms over the city, leaving the surroundings in a rain shadow.

There is a body of knowledge that suggests that the vegetation on green roofs can mitigate the Urban Heat Island through shading, insulation of the soil layer and evapotranspiration. Johannesburg inner city is built largely of reinforced concrete with flat roofs that are ideal for the installation of green roofs as no additional structural strengthening is required.

The problem is to model the existing Urban Heat Island in comparison with the natural terrain that would have characterised the pre-development state (taken as the temperature data from the Johannesburg Botanical Gardens). Various percentages of the area of the city with green roof can then be modelled to determine an optimal area under green roof in the city.

Problem 6. Chessboard Waves

Industry: Shipping and Tourism

Industry Representative: Thama Duba, University of the Witwatersrand, Johannesburg

Problem Statement:

In the popular tourist destination of the Isle of Rhe in France there are a set of waves that beautifully display the force of nature at its most powerful. The wave system showcases a strangely shaped square pattern looking like a chessboard that is perfectly displayed on the surface of the ocean. This is actually because the island is in an area where two seas meet, this is called a cross sea. Since the two seas that intersect in this area have different weather patterns and weather systems, the interaction of these waves form squares.

The square waves caused by cross seas are very dangerous. This is because the rip currents and riptides are stronger than average in this phenomenon. The square waves can become a danger to swimmers and surfers and have caused boating accidents and ship wrecks.

The study group is asked to answer the following questions:

1. What mechanism generates a square wave; for example, what type of waves would interact to form square waves?

2. What mathematical model would best describe a chessboard wave? Is it a deep-water wave or a shallow water wave?

3. What are the propagation properties of these waves?

4. What makes these waves dangerous?

5. Why is this phenomenon not being spotted at Cape Point?